

Post-critical *SsPmp* and its applications to Virtual Deep Seismic Sounding (VDSS)—1: sensitivity to lithospheric 1-D and 2-D structure

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Accepted 2018 July 24. Received 2018 July 3; in original form 2018 March 12

SUMMARY

Virtual Deep Seismic Sounding (VDSS) has recently emerged as a novel method to image the Moho and potentially other lithospheric boundaries. The behaviour of *SsPmp*, the post-critical reflection phase at the Moho that is utilized in VDSS, is rich with complexities not yet widely considered. Here, motivated by observations from the Ordos Plateau in North China, we use synthetic seismograms computed with a broad range of 1-D models to evaluate how different parts of the lithosphere along the ray path of *SsPmp* affect its phase, amplitude and arrival time. Our findings include: (1) When the crust–mantle boundary is a sharp discontinuity, the *SsPmp* phase shift relative to the direct *S* wave is controlled by lower-crustal V_p , upper-mantle V_p and ray parameter. This property indicates the possibility of using *SsPmp* to constrain V_p in the lower crust and uppermost mantle. (2) When the crust–mantle boundary is a velocity-gradient zone, *SsPmp* arrival times vary as different functions of ray parameter from cases with a sharp crust–mantle boundary, because different rays turn at different depths. This feature allows measurement of the vertical velocity gradient in the crust–mantle transition zone with *SsPmp*. (3) When the virtual source (location of *S*-to-*P* conversion at the free surface) is in a sedimentary basin, *SsPmp* amplitude can be significantly reduced due to low *S*-to-*P* reflected energy at the virtual source. This may cause the absence of *SsPmp* despite appropriate source–receiver geometry. In addition to 1-D models, we further conduct 2-D waveform modelling and find that the *SsPmp* arrival time relative to direct *S* is not only controlled by crustal thickness at the reflection point but also by lateral variation of V_s beneath the virtual source and receiver. Therefore, in areas with significant lateral heterogeneity in the lithosphere the accuracy of crustal-thickness measurements from *SsPmp* arrival times depends on our knowledge of the variability of lithospheric structure across a broad region.

Key words: Body waves; Wave scattering and diffraction; Crustal imaging; Crustal structure.

1 INTRODUCTION

For a quarter-century the seismic community has relied heavily on receiver function (RF) techniques, which use teleseismic *P*-to-*S* (*Ps* or *PRF*) or *S*-to-*P* (*Sp* or *SRF*) converted phases to image discontinuities in the lithosphere (Fig. 1; e.g. Langston 1979; Owens & Taylor 1984; Kind *et al.* 1995). In recent years, Virtual Deep Seismic Sounding (VDSS) has emerged as a novel method to image the crust–mantle boundary (CMB) and has been successfully applied to multiple data sets from different areas to yield Moho depths independent from RF results (e.g. Tseng *et al.* 2009; Yu *et al.* 2012, 2016; Liu *et al.* 2015; Kang *et al.* 2016; Parker *et al.* 2016). Here we use *CMB* to denote the geological transition from crust to mantle,

whether abrupt or a ‘velocity-gradient zone’ spanning many kilometres; and *Moho* to denote a seismological inference about this transition, typically given as a single depth. Post-critical *SsPmp*, the seismic phase used in VDSS, originates when upcoming teleseismic *S* waves convert to downgoing *P* waves at the free surface (the virtual source), and then undergo post-critical reflection at or within the CMB and finally reach the receiver (Fig. 1). Hereafter we use *SsPmp* to refer to post-critical *SsPmp* for simplicity. The arrival time of *SsPmp* relative to the direct *S* arrival (or *Ss*), hereafter T_{VDSS} , is used to estimate the Moho depth at the reflection point, normally assumed to be the midpoint between the virtual source and the receiver. Because VDSS uses teleseismic events, the incident *S* wave can be approximated as a plane wave with constant ray parameter.

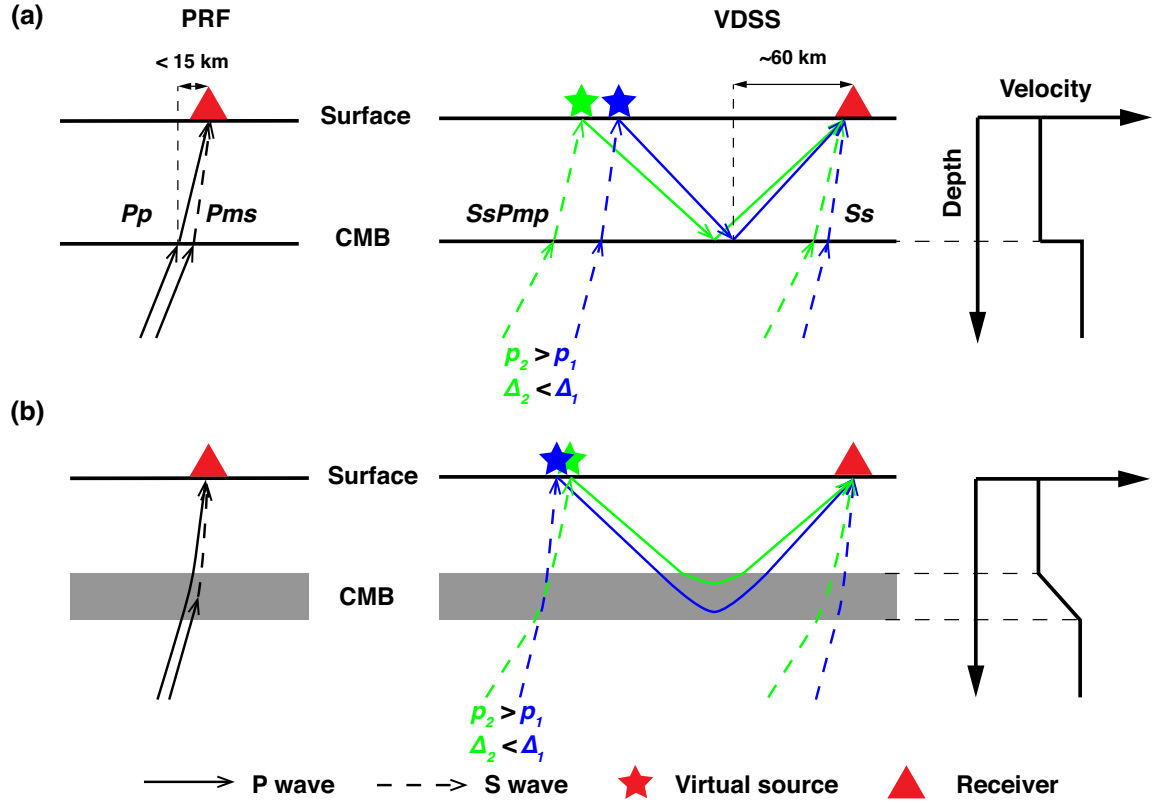


Figure 1. Comparison between ray paths for P receiver functions (PRFs; left column) and Virtual Deep Seismic Sounding (VDSS; right column) for (a) an abrupt velocity discontinuity at the crust–mantle boundary (CMB) and (b) a velocity-gradient zone at the CMB. For VDSS, blue and green colours are for earthquakes with epicentral distance Δ_1 and Δ_2 (or ray parameter p_1 and p_2), respectively.

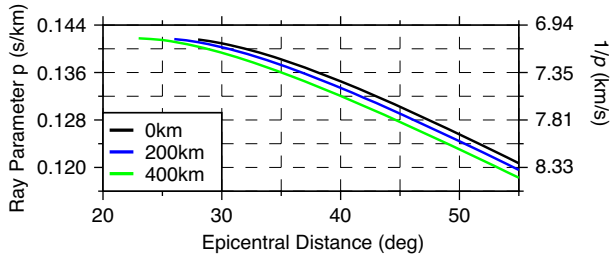


Figure 2. Ray parameter of teleseismic S wave (S_s) as a function of epicentral distance, for focal depths at 0, 200 and 400 km. Ray parameters are calculated using the IASP91 reference model (Kennett & Engdahl 1991). Note that ray parameters do not exceed 0.143 s km^{-1} in order to avoid triplication at the bottom of the mantle transition zone.

To enable post-critical reflection of $SsPmp$ at the Moho, appropriate epicentral distances are $\sim 30^\circ$ – 50° (Yu *et al.* 2016), corresponding to ray parameter $p = \sim 0.1256$ – 0.1409 s km^{-1} for a focal depth of 0 km ($1/p = \sim 7.10$ – 7.96 km s^{-1} ; Kennett 1991) and slightly smaller p for larger focal depths (Fig. 2). Due to post-critical reflection at the Moho, $SsPmp$ usually has amplitude comparable to S_s (Fig. 3b), making it possible to construct a seismic profile with a single event recorded by an array (Figs 4b and c). Because $SsPmp$ undergoes post-critical reflection at the Moho, its phase is changed relative to S_s (Aki & Richards 1980a; Fig. 3b; hereafter referred to as phase shift for simplicity and denoted Φ_{VDSS}), making it difficult to directly pick T_{VDSS} from raw records. In order to establish a robust measure of $SsPmp$ amplitude, we first separate vertical- and radial-component seismograms into pseudo- P (motion associated

with incoming P waves) and pseudo- S (motion associated with incoming S waves) components (hereafter P and S for simplicity) with a particle-motion analysis algorithm (Yu *et al.* 2013) so that Ss and $SsPmp$ appear only on S and P components, respectively (Fig. 3b). We then compute the envelope functions of Ss and $SsPmp$ and define the ratio between the peak values of $SsPmp$ and Ss envelope functions as A_{VDSS} (Fig. 3b). Because envelope function is independent of phase shift, A_{VDSS} is a robust measure of $SsPmp$ amplitude. For the simple single-layer crust model in Fig. 3(a), $A_{VDSS} = 0.86$. A_{VDSS} is not equal to 1 due to energy lost (1) at the virtual source where incident S waves generate reflected S waves and (2) at the Moho where downgoing P waves generate reflected and transmitted S waves. For simplicity, in most of the following analyses when variation in $SsPmp$ amplitude is qualitatively shown, we normalize $SsPmp$ with the peak of Ss waveforms and use it as a proxy for A_{VDSS} . However, we note that whenever quantitative analysis of $SsPmp$ amplitude is required, A_{VDSS} should be calculated (discussed further in Section 3.3.2 below).

Most previous studies (e.g. Tseng *et al.* 2009; Tian *et al.* 2015; Yu *et al.* 2016) used 1-D waveform fitting to measure T_{VDSS} , and then found the trade-off relation between crustal V_p and thickness H using the equation:

$$T_{VDSS} = 2H \sqrt{\frac{1}{V_p^2} - p^2}, \quad (1)$$

where p is the ray parameter of the incident S wave. In the process of waveform fitting, the upper mantle is commonly assumed to be a half-space with V_p fixed *a priori* (e.g. Tian *et al.* 2015; Yu

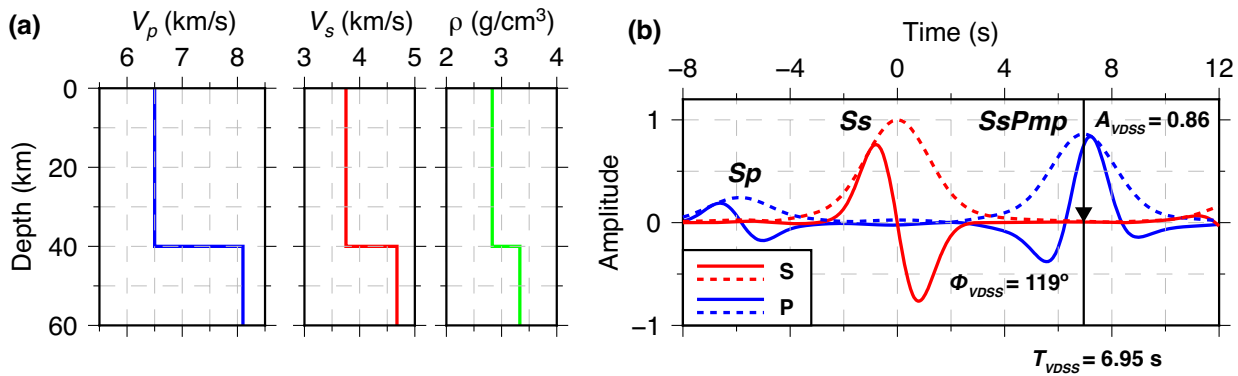


Figure 3. Synthetic post-critical $SsPmp$ waveforms. (a) 1-D velocity and density structure of a homogeneous crust over a half-space mantle used to compute waveforms in (b). (b) P - and S -component waveforms (solid) and their envelope functions (dashed) computed using models in (a) and aligned to Ss arrival. The ray parameter of the incident S wave is $p = 0.127 \text{ s km}^{-1}$ ($1/p = 7.87 \text{ km s}^{-1}$). The source time function is a first-order Hermitian function with a dominant frequency of $\sim 0.25 \text{ Hz}$. The Ss arrival is picked at the zero-crossing of the source wavelet or the peak of the envelope function. The waveforms are normalized by the peak amplitude of the Ss envelope function, so that A_{VDSS} is clearly shown. Black triangle marks the theoretical $SsPmp$ arrival time relative to Ss (T_{VDSS}). The large amplitude and phase shift of $SsPmp$ are due to post-critical reflection at the Moho.

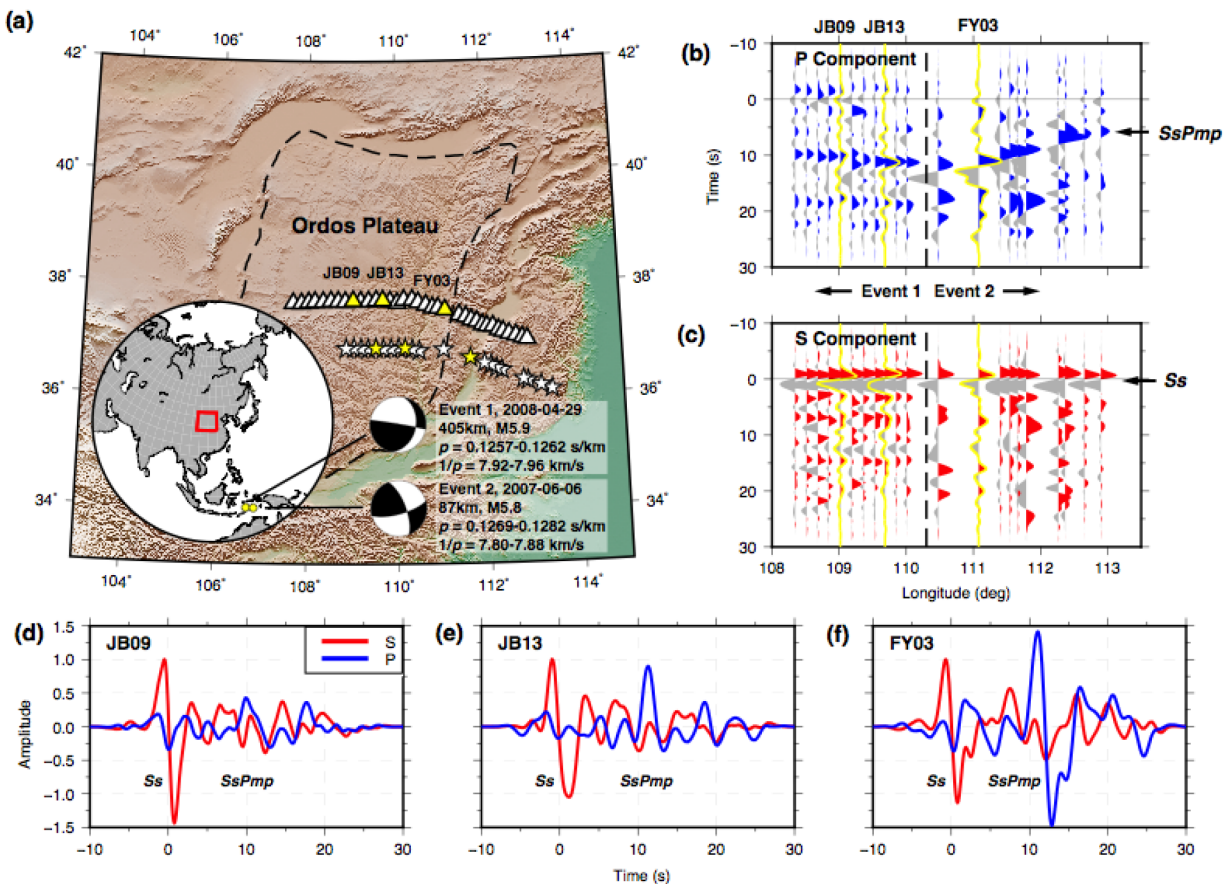


Figure 4. $SsPmp$ observations from the Ordos Plateau, North China. (a) Broad-band seismic stations (triangles) and their corresponding virtual sources (stars) for two closely spaced deep-focus Banda Sea earthquakes, Events 1 and 2. Virtual source locations are calculated using a crustal thickness of 40 km and $V_p = 6.3 \text{ km s}^{-1}$. Yellow triangles and stars correspond to yellow-highlighted traces in (b) and (c) shown enlarged in (d) and (e). Lower-left inset shows location of Events 1 and 2 (yellow circles). The range of ray parameters (p) shown is for the entire array. (b) A composite profile of P component of the S -wave train from Event 1 (recorded west of 110.3°E) and Event 2 (recorded east of 110.3°E). (c) The same as (b) but for the SV component. P - and SV -component waveforms are both aligned by the Ss phase (0 s, picked at the zero-crossing) and normalized by its maximum amplitude. Dashed vertical lines separate the parts of the array that recorded each of the two events. (d–f) Enlargements of yellow-highlighted traces in (b) and (c). Note the significant difference in $SsPmp$ amplitude between JB09 and JB13, and the difference in Φ_{VDSS} between JB13 ($\sim 90^\circ$ phase shift from Ss) and FY03 ($\sim 0^\circ$ phase shift).

et al. 2016). Recently, Parker *et al.* (2016) proposed picking T_{VDSS} directly from the envelope function of *SsPmp* because the envelope function is independent of the phase shift. For the purposes of waveform fitting, previous VDSS studies have commonly assumed that the CMB is a discontinuity in V_p , since Tseng *et al.* (2009) and Yu *et al.* (2012) have shown that *SsPmp* waveforms appear similar whether the CMB is a velocity-gradient zone or a discontinuity. Multiples (reverberations) generated in sedimentary basins severely contaminate RF waveforms and have presented a long-standing challenge to the utility of RFs (e.g. Langston 2011; Tao *et al.* 2014; Yu *et al.* 2015; Bao & Niu 2017; Guan & Niu 2017). In contrast, the manner in which sedimentary basins affect *SsPmp* waveforms is not widely understood. Parker *et al.* (2016) showed that *SsPmp* varies with velocity variation within basins, and Ferragut *et al.* (2017) searched for but did not find effects from a <1 km-thick basin.

Here we present as motivation a data set from the Ordos Plateau in North China in which significant changes in *SsPmp* phase shift, amplitude and traveltime (Φ_{VDSS} , A_{VDSS} and T_{VDSS}) are observed across the recording array. We first use synthetic waveforms to demonstrate that, when the CMB is a sharp discontinuity, Φ_{VDSS} decreases from $\sim 180^\circ$ to $\sim 0^\circ$ as the lower-crustal V_p (hereafter V_p^{lc}) or upper-mantle V_p (hereafter V_p^{um}) or ray parameter p increases. Additionally, when p is small ($1/p$ is close to V_p^{um}) Φ_{VDSS} is more sensitive to V_p^{um} than V_p^{lc} , whereas when p is large ($1/p$ is close to V_p^{lc}) Φ_{VDSS} is more sensitive to V_p^{lc} than V_p^{um} . We next show that when the CMB is a velocity-gradient zone with thickness up to the dominant wavelength of incident *S* wave (*Ss*), neither Φ_{VDSS} nor A_{VDSS} varies significantly with p , but T_{VDSS} varies as different functions of p from the case with a sharp CMB, because rays with different ray parameters turn at different depths within the velocity-gradient zone. This feature should make VDSS an ideal method to study CMB structure in areas such as the Colorado Plateau, where a sharp Moho is not observed by traditional methods based on pre-critical reflections and conversions (Hauser & Lundy 1989; Shen *et al.* 2013). We also demonstrate that although A_{VDSS} is largely insensitive to CMB structure, it is significantly decreased by low near-surface velocity at the virtual source, which may explain cases of the absence of *SsPmp* from records with otherwise proper observation geometries. Finally, we use 2-D synthetic seismograms to show that T_{VDSS} is affected not only by crustal thickness at the reflection point but also by heterogeneity in V_s structure beneath the virtual source and receiver, for instance lateral variation in lithospheric thickness, which must be accounted for when using T_{VDSS} to estimate crustal thickness. Our analyses qualitatively explain the variation of Φ_{VDSS} and A_{VDSS} in our example from the Ordos Plateau. In our companion paper (Liu *et al.*, unpublished manuscript, hereafter ‘Part 2’), we will discuss practical approaches to inferring lithospheric structure from *SsPmp* observations.

2 DATA

The Ordos Plateau in North China (Fig. 4a) is a site of considerable interest, as it is the surviving remnant of the Archean North China Craton (Liu *et al.* 1992). It is unknown why this region preserved its lithospheric root while the eastern part of the craton underwent significant modification (decratonization) in Mesozoic time (Menzies *et al.* 1993; Griffin *et al.* 1998; Gao *et al.* 2008). To study the lithospheric structure of the Ordos Plateau, a W–E linear seismic

array was deployed across the Ordos Plateau at $\sim 37.5^\circ\text{N}$ during 2006–2008 by Peking University (Yu *et al.* 2012), and recorded abundant intermediate to deep seismicity in the west Pacific subduction zones at epicentral distances of 30° – 50° , making the data set ideal for observing post-critical *SsPmp* (Fig. 4a). Because the western and eastern parts of the array were deployed in two different time periods, we use two nearby events (~ 400 km separation) in the Banda Sea to make a single record section (Figs 4b and c). The radial- and vertical-component traces are bandpass filtered between 0.04 and 0.5 Hz, separated into *P* and *SV* components with a particle-motion analysis algorithm (Yu *et al.* 2013) and aligned by their direct *S* (*Ss*) arrival times. The *Ss* arrival times are picked at the zero-crossings of the *Ss* wavelets, which closely resemble a first-order Hermitian wavelet (first derivative of Gaussian wavelet; Figs 4b–f), because for a first-order Hermitian wavelet the peak of its envelope function is at its zero-crossing and does not change after an arbitrary phase shift. The waveforms are normalized to the peak amplitudes of the corresponding direct *S* arrivals.

We observe significant variation in *SsPmp* amplitude across the array. For instance, the *SsPmp* amplitude of JB09 is only about half of JB13 (Figs 4d and e), despite a station separation of only ~ 60 km. Because the traces are normalized by the peak amplitudes of *Ss*, the effects of source radiation pattern are removed (and should be minor anyway since the aperture of the array is small compared to the epicentral distance). The rapid change of *SsPmp* amplitude across the array thus must be due to structure near the stations. Φ_{VDSS} also varies rapidly across the array. At JB13, *SsPmp* has a $\sim 90^\circ$ phase advance relative to *Ss* (Fig. 4e), as commonly assumed for post-critical *SsPmp* (Tseng *et al.* 2009). However, only ~ 120 km distant, at FY03 the *SsPmp* waveform closely resembles *Ss*, indicating a $\sim 0^\circ$ phase shift (Fig. 4f). Φ_{VDSS} is unrelated to the source and should be primarily controlled by structure close to the stations. *SsPmp* amplitude and Φ_{VDSS} are crucial properties that need to be correctly modelled in order to measure T_{VDSS} . If the signal-to-noise ratio of *SsPmp* is very small, it will be difficult to measure its arrival time. Even when *SsPmp* amplitude is high, as for JB13 and FY03, improperly modelling the phase of *SsPmp* will cause up to ~ 1 s error in T_{VDSS} , which corresponds to ~ 5 km error in the resulting crustal thickness estimation, a non-trivial error in most applications. The phenomenon of significant spatial variation of *SsPmp* amplitude and Φ_{VDSS} has not been systematically explored by previous studies. Another unexpected feature of the Ordos data set is that T_{VDSS} varies significantly across the array (Fig. 4a). Using the arrival time of the strongest peaks on the *P* component as an approximation of T_{VDSS} (equivalent to assuming a 90° phase shift), we find that T_{VDSS} reaches its maximum of ~ 12 s at $\sim 110.5^\circ\text{E}$, nominally corresponding to a crustal thickness of ~ 60 km, and then decreases rapidly eastwards, reaching ~ 6 s at $\sim 113^\circ\text{E}$, which corresponds to a crustal thickness of ~ 30 km (Yu *et al.* 2012). This change in traveltime is far greater than possible phase delays that are limited to 0° – 180° , or ~ 2 s for these ~ 4 s period arrivals. The unexpectedly deep Moho was previously interpreted as preservation of a ~ 20 km-thick, mafic lower-crustal layer beneath the Ordos Plateau, relict from craton formation (Yu *et al.* 2012). However, it has recently been recognized that lateral variation in V_s structure can affect T_{VDSS} (Yu *et al.* 2016), and thus it is important to quantify this effect. In the following sections, we use synthetic seismograms to examine possible lithospheric structures that may affect A_{VDSS} , Φ_{VDSS} and T_{VDSS} .

3 SYNTHETIC TESTS

3.1 Sensitivity to V_p across the crust–mantle boundary

In this section, we first assume that the lithosphere consists of three homogeneous layers separated by abrupt discontinuities: the upper crust, lower crust and upper mantle (Figs 5a and 6a). The *SsPmp* waveform is affected by V_p of the lower crust and upper mantle, therefore we first vary V_p^{lc} with fixed V_p^{um} (Fig. 5a), and then vary V_p^{um} with V_p^{lc} fixed (Fig. 6a), to test the sensitivity of the *SsPmp* waveform to the two parameters. As V_p varies, we vary V_s to keep Poisson's ratio ν constant at 0.25, and vary density to satisfy the Nafe–Drake relation (eq. 1 in Brocher 2005). We compute radial- and vertical-component waveforms using the reflectivity algorithm in a layered medium (Randall 1989), in which the incident *S* wave is set with ray parameter $p = 0.125$ or 0.130 s km⁻¹ ($1/p = 8.00$ or 7.69 km s⁻¹) in order to test the potential effects of p . The radial- and vertical-component waveforms are then separated into *S* and *P* components. To separately examine reflections at the CMB and intracrustal interfaces, which usually interfere with each other, we also compute synthetic wavelets for the reflections separately by directly applying the reflection coefficients at the corresponding interfaces to the incident wavelet (Aki & Richards 1980b; Figs 5c–f, 6c and d). The only difference between the wavelets computed with this approach and the ones given by the reflectivity method is the *S*-to-*P* reflection coefficient at the virtual source, which is constant across all models and, thus, does not affect the comparison between the models. The source time function is assumed to be a first-order Hermitian wavelet (first derivative of a Gaussian wavelet) with a half width of ~ 2 s, which has similar shape and frequency to observed *Ss* of deep or intermediate-depth earthquakes (Fig. 4). We note that deconvolving the source wavelet (in this case the first-order Hermitian wavelet) from the radial and vertical components can reduce the source time function to simpler forms (e.g. Gaussian wavelets), which may reduce complexities of the waveforms and naturally align the traces (Yu *et al.* 2013; Parker *et al.* 2016). However, deconvolution is a non-unique inversion that can introduce artefacts, so here we avoid it in order to more clearly show the underlying physics of *SsPmp*.

Fig. 5(b) shows synthetic seismograms computed with V_p^{lc} ranging from 7.0 to 7.6 km s⁻¹, V_p^{um} fixed at 8.1 km s⁻¹ and $p = 0.125$ km s⁻¹. This range of V_p^{lc} is characteristic of mafic continental lower crust (Christensen & Mooney 1995). As V_p^{lc} increases, *SsPmp* arrives earlier (T_{VDSS} decreases) and becomes closer to the reflection phase at the top of the lower crust (*SsPip* with *i* standing for intracrustal interface; Fig. 5b), with little change in its amplitude or Φ_{VDSS} (Fig. 5d). The amplitude of *SsPip* grows as V_p^{lc} increases (Fig. 5c), interfering with *SsPmp* and distorting its waveform (Fig. 5b). The different responses of *SsPip* and *SsPmp* to velocity changes indicate their different natures. As a pre-critical reflection, the amplitude of *SsPip* grows as the velocity contrast at the intracrustal interface increases. In contrast, as a post-critical reflection phase, the amplitude of *SsPmp* (a proxy for A_{VDSS}) stays constant despite changes of velocity contrast across the Moho, because all incident energy is reflected upwards at the CMB (V_p^{um} exceeds the ray parameter p). To test the effects of ray parameter, we compute the *SsPmp* and *SsPip* wavelets for $p = 0.130$ s km⁻¹. The *SsPmp* wavelet for $p = 0.130$ s km⁻¹ shows significantly smaller Φ_{VDSS} for every V_p^{lc} value than for $p = 0.125$ s km⁻¹ (Figs 5d and f). In addition, we observe a significant decrease of Φ_{VDSS} with increasing V_p^{lc} for $p = 0.130$ s km⁻¹ (Fig. 5f). These observations indicate that ray parameter can have profound effects

on Φ_{VDSS} and its sensitivity to V_p^{lc} . As expected, *SsPip* amplitude increases with increasing p due to a wider incident angle at the intracrustal discontinuity (Fig. 5e).

Fig. 6 shows the sensitivity of *SsPmp* waveforms to V_p^{um} . We compute synthetic seismograms using models with V_p^{um} ranging from 8.1 to 8.7 km s⁻¹ and V_p^{lc} fixed at 7.0 km s⁻¹ (Fig. 6a). The range of V_p^{um} used here covers much of the observed range for the continental uppermost mantle (Christensen & Mooney 1995), and also ensures post-critical reflection of *SsPmp* at the CMB for our chosen ray parameters ($1/p = 7.69$ and 8.00 km s⁻¹). When $p = 0.125$ s km⁻¹, our synthetic seismograms show strong dependence of Φ_{VDSS} on V_p^{um} (Fig. 6b). When $V_p^{um} = 8.1$ km s⁻¹, Φ_{VDSS} is close to 180° (corresponding to the source wavelet multiplied by -1) and the waveform is similar to the pre-critical reflection. As V_p^{um} increases, Φ_{VDSS} first decreases to $\sim 90^\circ$ and then further drops to below 90° (Figs 6b and c). Another important observation is that despite the significant change in phase, *SsPmp* amplitude stays almost the same as V_p^{um} increases, because all incident energy is post-critically reflected. When p increases to 0.130 s km⁻¹, Φ_{VDSS} decreases and becomes less sensitive to changes of V_p^{um} (Fig. 6d). This observation shows the significant effects of ray parameter on Φ_{VDSS} and its sensitivity to V_p^{um} . Because Φ_{VDSS} (the *SsPmp* waveform) is sensitive to V_p^{um} , one must know V_p^{um} in order to correctly measure T_{VDSS} . An example is shown in Fig. 6(e), in which we assume the *SsPmp* wavelet to be phase shifted by 90° from *Ss*, and cross-correlate it with synthetic waveforms in Fig. 6(b) to find T_{VDSS} . The resulting T_{VDSS} for each model is then converted to estimated crustal thickness (Moho depth) using eq. (1) with the true average crustal V_p . We observe that a 90° phase shift is only a good approximation when $V_p^{um} = \sim 8.47$ km s⁻¹, whereas for $V_p^{um} = 8.1$ km s⁻¹ the estimated Moho depth is ~ 3 km larger than the true value (Fig. 6e). This example highlights the importance of using the correct V_p^{um} in measuring T_{VDSS} and estimating Moho depth.

To further explore the effects of ray parameter p on Φ_{VDSS} and its sensitivity to V_p^{lc} and V_p^{um} , we calculate Φ_{VDSS} as functions of V_p^{lc} and V_p^{um} for $p = 0.125$ and 0.130 s km⁻¹ ($1/p = 8.00$ and 7.69 km s⁻¹) by taking the phases of the complex *P*-to-*P* reflection coefficients at the Moho (Aki & Richards 1980a,b). Our first-order observation is that Φ_{VDSS} decreases for all V_p^{lc} and V_p^{um} when p increases from 0.125 to 0.130 s km⁻¹ (Fig. 7), which agrees with our synthetic waveforms (Figs 5d, f, 6c and d). In addition, as p increases from 0.125 to 0.130 s km⁻¹, Φ_{VDSS} becomes less sensitive to V_p^{um} and more sensitive to V_p^{lc} [contours in Fig. 7 are steeper in (b) than in (a)], which also agrees with our synthetic waveforms (Figs 5d, f, 6c and d). Moreover, for a fixed p , Φ_{VDSS} decreases with increasing V_p^{um} and V_p^{lc} , and as Φ_{VDSS} decreases it becomes less sensitive to V_p^{um} and more sensitive to V_p^{lc} (contours steepen from lower left to upper right in both Figs 7a and b). Because $V_p^{lc} < 1/p < V_p^{um}$, increasing p while fixing V_p^{um} and V_p^{lc} or increasing V_p^{um} and V_p^{lc} while fixing p are both equivalent to moving $1/p$ closer to V_p^{lc} and farther from V_p^{um} . Therefore, Fig. 7(a) shows that when $1/p$ is close to V_p^{um} , Φ_{VDSS} is more sensitive to V_p^{um} than V_p^{lc} (~ 10 times more sensitive to V_p^{um} in bottom left of Fig. 7a as indicated by the reference line), which can be understood by considering that as $1/p$ approaches V_p^{um} , the reflection coefficient approaches the pre-critical value of 180° (sign reverse) phase shift regardless of changes in V_p^{lc} . In contrast, Fig. 7(b) shows that when $1/p$ is close to V_p^{lc} , Φ_{VDSS} is more sensitive to V_p^{lc} than V_p^{um} (~ 5 times more sensitive to V_p^{lc} in top right of Fig. 7b as indicated by the reference line), which can be understood by considering that as $1/p$ approaches V_p^{lc} , V_p^{lc}

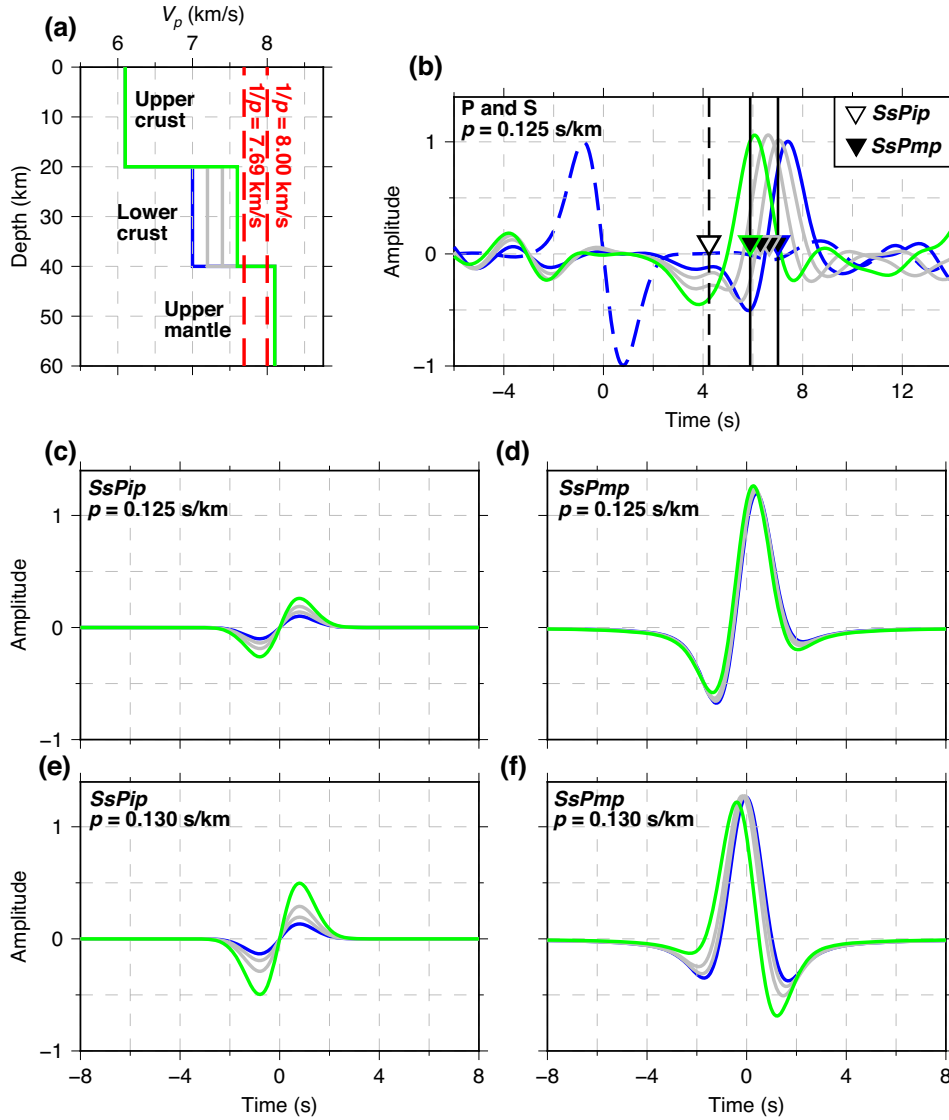


Figure 5. *SsPmp* waveforms computed for ray parameter $p = 0.125$ and 0.130 s km^{-1} and V_p^{lc} ranging from 7.0 to 7.8 km s^{-1} . (a) V_p models used to compute the *SsPmp* synthetic waveforms. Blue: $V_p^{\text{lc}} = 7.0 \text{ km s}^{-1}$. Green: $V_p^{\text{lc}} = 7.6 \text{ km s}^{-1}$. Grey: intermediate models. Red dashed lines: critical V_p of the incident waves. (b) Blue, grey and green curves: P -component synthetic waveforms computed with $p = 0.125 \text{ s km}^{-1}$, coloured as in (a). Dashed blue curve: S -component synthetic waveform computed with $V_p^{\text{lc}} = 7.0 \text{ km s}^{-1}$. Black triangles: theoretical T_{VDSS} for each synthetic waveform. Open triangle: theoretical arrival time of the pre-critical intracrustal reflection (*SsPip*), constant for all models. (c and d) *SsPip* and *SsPmp* wavelets, respectively computed with $p = 0.125 \text{ s km}^{-1}$, coloured as in (b). (e and f) The same as (c) and (d) except for ray parameter $p = 0.130 \text{ s km}^{-1}$. Note V_p^{lc} has a stronger effect on Φ_{VDSS} for larger ray parameter.

controls where post-critical reflection occurs between the top of the lower crust and the CMB, and V_p^{um} has no effect. Cases with $1/p$ intermediate between V_p^{lc} and V_p^{um} lie between the two extremes.

3.2 Sensitivity to CMB thickness

To understand how CMB thickness affects *SsPmp* waveforms, we model the CMB as a transition from $V_p = 6.5 \text{ km s}^{-1}$, typical of crustal rocks, to $V_p = 8.1 \text{ km s}^{-1}$, typical of uppermost mantle rocks. As before, V_s and density are varied simultaneously with V_p to keep Poisson's ratio at 0.25 and to satisfy the Nafe–Drake relation. The ray parameter p is set as 0.127 s km^{-1} ($1/p = 7.87 \text{ km s}^{-1}$), in between the two extreme cases discussed above ($p = 0.125$ and 0.130 s km^{-1}). We model the transition zone as a stack of 1 km

thick layers and vary the transition thickness from 0 to 20 km below a fixed top at 40 km (Fig. 8a). Similar models have been used (Braile & Chiang 1986) to study Moho signatures in active source seismic reflection and refraction data, and to show that amplitude–distance characteristics of the P_n and P_mP phases can be used to estimate the thickness of the CMB transition zone. For post-critical *SsPmp*, as the CMB becomes thicker, there is little change in Φ_{VDSS} (Fig. 8b) until the thickness exceeds the dominant wavelength (Fig. 8c). As the CMB thickens, A_{VDSS} stays stable with a slight increase due to decreased pre-critical P -to- S reflection and transmission at the CMB when it becomes a gradient zone (Fig. 8c). This behaviour is distinct from that of pre-critical reflected and converted phases at the CMB, which become broader and weaker as the CMB thickens. The clear change in Fig. 8(b) is that T_{VDSS} becomes larger as the post-critical reflection occurs at greater depth, because in each case the incident

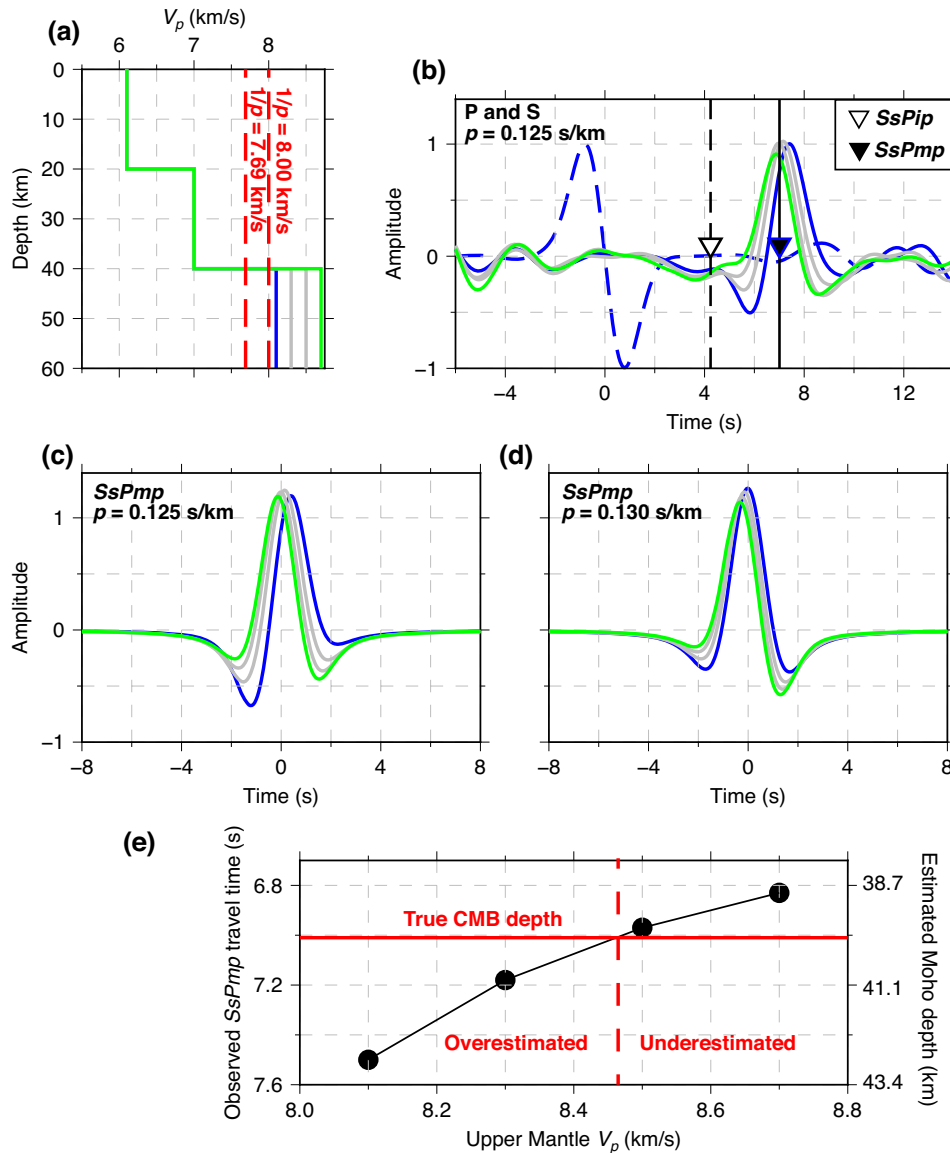


Figure 6. *SsPmp* waveforms computed for ray parameter $p = 0.125$ and 0.130 s km $^{-1}$ and V_p^{um} ranging from 8.1 to 8.7 km s $^{-1}$. (a) V_p models. Blue: $V_p^{\text{um}} = 8.1$ km s $^{-1}$. Green: $V_p^{\text{um}} = 8.7$ km s $^{-1}$. Grey: intermediate models. Red dashed line: critical V_p of the incident waves. (b) Blue, grey and green curves: P -component synthetic waveforms coloured as in (a). Dashed blue curve: S -component synthetic waveform for $V_p^{\text{um}} = 8.1$ km s $^{-1}$. Black (and open) triangles: theoretical T_{VDSS} (and $SsPip$), constant for all models. (c and d) *SsPmp* wavelets computed with $p = 0.125$ and 0.130 s km $^{-1}$, coloured as in (b). Note that V_p^{um} has a stronger effect on Φ_{VDSS} for smaller ray parameter. (e) Black dots: observed T_{VDSS} and estimated Moho depth for waveforms in (b) assuming a 90° phase shift of *SsPmp* relative to *Ss*. Solid red line: true CMB depth of 40 km.

wave turns within the CMB where $V_p = 1/p$, so the turning depth becomes larger as the CMB thickness increases (Fig. 8a).

Fig. 9 compares the responses of PRF and VDSS to different CMB thicknesses (Figs 9a and b). The model in Fig. 9(b) is identical to the model with a 20 km-thick CMB in Fig. 8(a), whereas Fig. 9(a) is the 0 km-thick CMB model in Fig. 8(a) with the discontinuity shifted from 40 to 50 km, so that the centres of the CMBs are at the same depth in Figs 9(a) and (b). When the incident waves are plane P waves with ray parameters in the range 0.04 – 0.08 s km $^{-1}$, the computed PRFs are strong and narrow for a discontinuous CMB (Fig. 9c), but broad and weak for a transitional CMB (Fig. 9d; even the strongest Pms on the PRFs shown here is only ~ 20 per cent the amplitude of the first arrival). As the CMB thickens, high-frequency components of the incident P waves with wavelengths shorter than

the scale of the CMB become insensitive to it, eliminating high-frequency components of Pms . The Pms waveforms stay almost unchanged as the ray parameter varies from 0.04 to 0.08 s km $^{-1}$ for both a sharp CMB (Fig. 9c) and a broad CMB (Fig. 9d). Theoretical Pms arrival times (calculated assuming that conversion occurs at the centre of the CMB) closely match the centres of observed Pms , and are very similar at the same ray parameter for both sharp and broad CMBs (Figs 9c and d). The above behaviours would also be observed for pre-critical Pmp and $SsPmp$ reflections.

In contrast, post-critical *SsPmp* is a strong phase with similar amplitude to *Ss* no matter the CMB is a discontinuity or a broad transition zone (Figs 9e and f, 8b and c). The reason that post-critical *SsPmp* remains strong as the CMB thickens is that all frequency components of the downgoing P wave are reflected (or turned) at

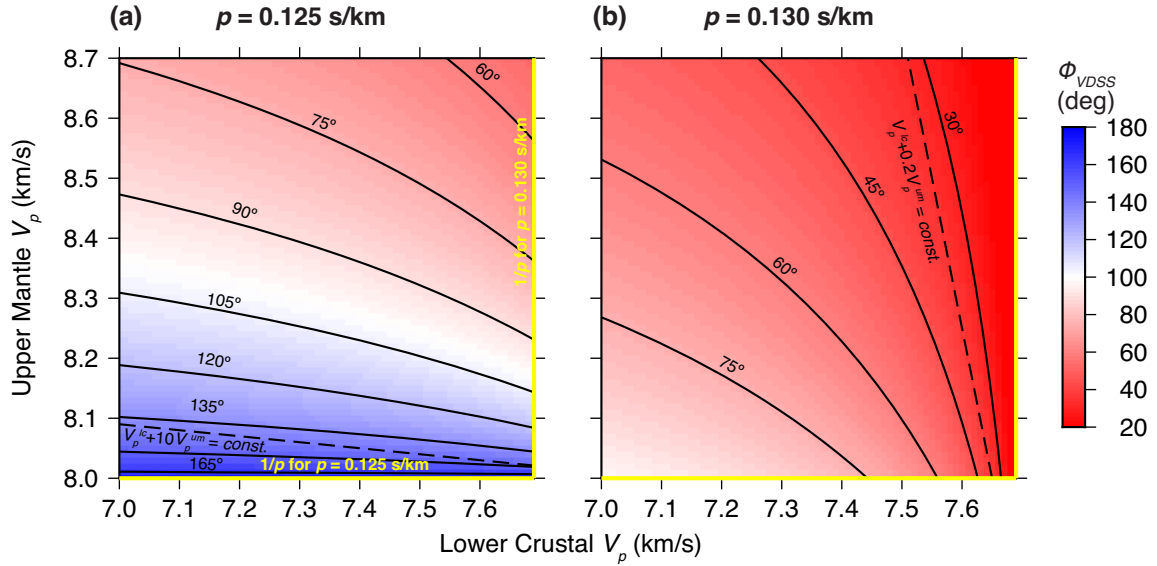


Figure 7. Φ_{VDSS} contoured at 15° intervals, as functions of V_p^{lc} and V_p^{um} for (a) $p = 0.125 \text{ s km}^{-1}$ and (b) $p = 0.130 \text{ s km}^{-1}$. V_s and densities are varied to keep a constant Poisson's ratio of 0.25, and to satisfy the Nafe–Drake relation (Brocher 2005), respectively. Two linear trade-off relations between V_p^{lc} and V_p^{um} are plotted in black dashed lines for reference. Yellow lines: critical velocities for $p = 0.125$ and 0.130 s km^{-1} . Note that when $1/p$ is close to V_p^{um} , Φ_{VDSS} is primarily sensitive to V_p^{um} [contours with low slopes, bottom of (a)], whereas when $1/p$ is close to V_p^{lc} , Φ_{VDSS} is primarily sensitive to V_p^{lc} [contours with high slopes, right side of (b)].

the depth where, for that ray parameter, V_p reaches the turning velocity. As a result, T_{VDSS} predicted using the reflection depth as the depth where the turning velocity is reached follows the moveout of *SsPmp* closely in the transitional model (Fig. 9f) and differs significantly from T_{VDSS} in the sharp CMB case (Fig. 9e).

When an incident plane wave turns in a medium with positive vertical velocity gradient, the turning wave will have a 90° phase shift relative to the incident wavelet (Shearer 1999). However, our synthetic waveforms show that, for CMB thickness $< 20 \text{ km}$, Φ_{VDSS} of the transitional models are similar to the discontinuity model with the same p (Figs 8b, 9e and f), instead of 90° . The reason is that the dominant wavelength of downgoing P wave in the CMB ($\sim 25 \text{ km}$ for a dominant frequency of $\sim 0.25 \text{ Hz}$) is similar to the CMB thickness ($< 20 \text{ km}$), causing Φ_{VDSS} to be insensitive to the presence of the transition zone. Since the transitional models have the same total velocity increase across the CMB as the discontinuity model, Φ_{VDSS} of the transitional models are similar to the discontinuity model at a given p . To further test our hypothesis, we computed Φ_{VDSS} for CMB thicknesses up to 60 km with the total velocity increase across the CMB being the same as the discontinuity model (Fig. 8c). The results show that as CMB thickness exceeds $\sim 20 \text{ km}$, Φ_{VDSS} starts to deviate significantly from the discontinuity model ($\Phi_{VDSS} = 119^\circ$), and gradually approaches 90° as CMB thickness further increases (Fig. 8c). This supports our hypothesis that Φ_{VDSS} is similar in both transitional and discontinuity models due to finite-frequency effects. In principal, higher frequency incident waves would show a more pronounced difference between Φ_{VDSS} produced by a sharp and a broad CMB, though recorded teleseismic S waves seldom have useful energy above 1 Hz . Therefore, we conclude that, for CMB thickness smaller than $\sim 25 \text{ km}$, no significant difference exists between Φ_{VDSS} caused by a sharp CMB and by a broad CMB, though the two models cause distinct moveout of T_{VDSS} with p (Fig. 9f).

3.3 Sensitivity to sedimentary basins

In the presence of sedimentary basins, we expect the waveforms of *SsPmp* to be affected by the thickness, velocity, Poisson's ratio ν (or V_p/V_s ratio) and velocity gradient of the sedimentary layer. We next test the sensitivity of *SsPmp* waveforms to sedimentary basins by varying each of these four parameters, one at a time (Figs 10a–d). Our reference basin model is a homogeneous layer with $V_p = 4.5 \text{ km s}^{-1}$, $\nu = 0.25$ and thickness $= 6 \text{ km}$. For simplicity, we assume the basin has uniform properties both at the virtual source and at the receiver (Fig. 10j). The radial- and vertical-component waveforms are separated into P and S components as in previous sections.

3.3.1 Sedimentary-layer thickness

To test the effects of sedimentary-layer thickness on *SsPmp* waveforms, we vary thickness from 0 km (basement at surface), 3 km (shallow basin) to 6 km (deep basin; Fig. 10a). Both basin cases have similar *SsPmp* amplitude, clearly lower than the basement case (Fig. 10e). *SsPmp* arrives increasingly later (T_{VDSS} increases) as the basin becomes deeper, because the basin has lower velocities than the basement (Fig. 10e). Also visible in the two basin cases are the seismic arrivals *Sbp* (S -to- P conversion phase at the basin–basement interface b) and *SsPbp* (pre-critical P reflection phase at the basin–basement interface after S -to- P reflection at the free surface; Fig. 10e), both delayed slightly from *Ss* (note that *PpPbp* has been used for attempted direct basin imaging, e.g. Yang *et al.* 2012). As the basin becomes thicker, *Sbp* and *SsPbp* are increasingly delayed from *Ss* (Fig. 10e). *SsbPmp* (S -to- P reflection at the basin–basement interface followed by post-critical reflection at the Moho) is not observed because the velocity contrast at the basin–basement interface is not large enough to generate strong S -to- P reflections (Fig. 10e).

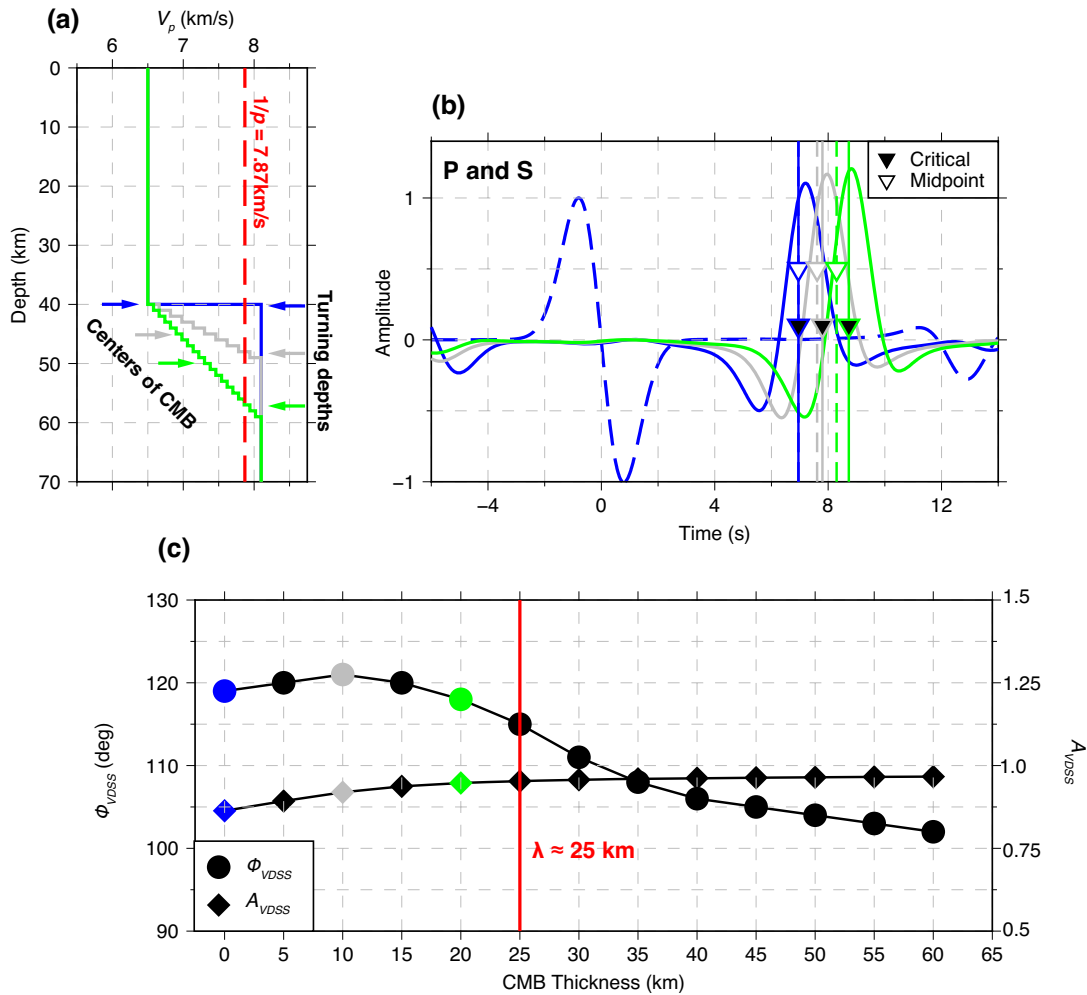


Figure 8. Effects of CMB thickness on *SsPmp* waveforms. (a) V_p models. Blue: CMB as a velocity discontinuity at 40 km depth. Grey: 10 km-thick CMB. Green: 20 km-thick CMB. V_p at the top and bottom of the CMB are fixed at 6.5 and 8.1 km s⁻¹. Red dashed line: critical V_p of the incident wave. (b) Blue, grey and green curves: *P*-component synthetic waveforms coloured as in (a). Dashed blue curve: *S*-component synthetic seismogram computed with the discontinuous CMB model plotted for reference (*Ss* is essentially identical for all three models). Solid vertical lines and black triangles with blue, grey and green edges: theoretical T_{VDSS} of the models coloured as in (a), calculated for the depths at which $V_p = 7.87$ km s⁻¹, that is, at which V_p equals the turning velocity ($1/p$). Dashed vertical lines and open triangles with blue, grey and green edges: theoretical T_{VDSS} calculated for the depths to the centre of the CMB. (c) Φ_{VDSS} (circles) and A_{VDSS} (diamonds) as functions of CMB thickness. V_p at the top and bottom of the CMB are fixed at 6.5 and 8.1 km s⁻¹. Blue, grey and green: models coloured as in (a). Red vertical line: dominant wavelength λ of *P* waves with dominant frequency 0.25 Hz in the CMB. Note that as CMB thickness becomes larger than λ , Φ_{VDSS} deviates significantly from the value with a velocity discontinuity.

The reduced *SsPmp* amplitude in the presence of sedimentary cover has profound significance, as none of the parameters previously examined (V_p^{lc} , V_p^{um} and CMB thickness) have significant effects on *SsPmp* amplitude, thus cannot explain observations of highly variable *SsPmp* amplitude (Figs 4d and e). We interpret the low-*SsPmp* amplitude in the presence of a sedimentary basin as caused by the low V_s at the virtual source, which reduces the incident angle of the incoming *S* wave at the free surface and lowers the reflection coefficient between the incident *S* wave and reflected *P* wave (in the extreme case, a vertically incident *S* wave produces no reflected *P* wave). Because it is the reflected *P* wave at the free surface that is later reflected at the Moho to form *SsPmp*, the sedimentary layer reduces *SsPmp* amplitude. In other words, the ‘efficiency’ of the virtual source decreases due to the sedimentary basin at the virtual source. Because the dominant wavelength of the incident *S* wave is ~ 10 km in the sedimentary layer, significantly larger than 3 and 6 km, the increase of basin thickness from 3 to 6 km

does not greatly increase the affected spectrum. Therefore, *SsPmp* amplitude does not vary significantly when the basin deepens from 3 to 6 km (Fig. 10e).

3.3.2 Sedimentary layer velocities

To test the effects of sedimentary layer velocities on *SsPmp* waveforms, we vary V_p in our homogeneous basin from 4.5 km s⁻¹ (high- V_p case), 3.5 km s⁻¹ (intermediate V_p), to 2.5 km s⁻¹ (low- V_p case; Fig. 10b). Because we hold v constant, V_s of the basin changes accordingly. As sedimentary V_p drops, we observe a dramatic decrease in *SsPmp* amplitude (Fig. 10f). In the high- V_p case, *SsPmp* is still the dominant phase, whereas in the intermediate- and low- V_p cases A_{VDSS} becomes so low that it is hard to distinguish it from other basin phases (Fig. 10f). Another important observation is that in the low- V_p case a strong *P* phase arrives at ~ 4.5 s, ~ 5 s

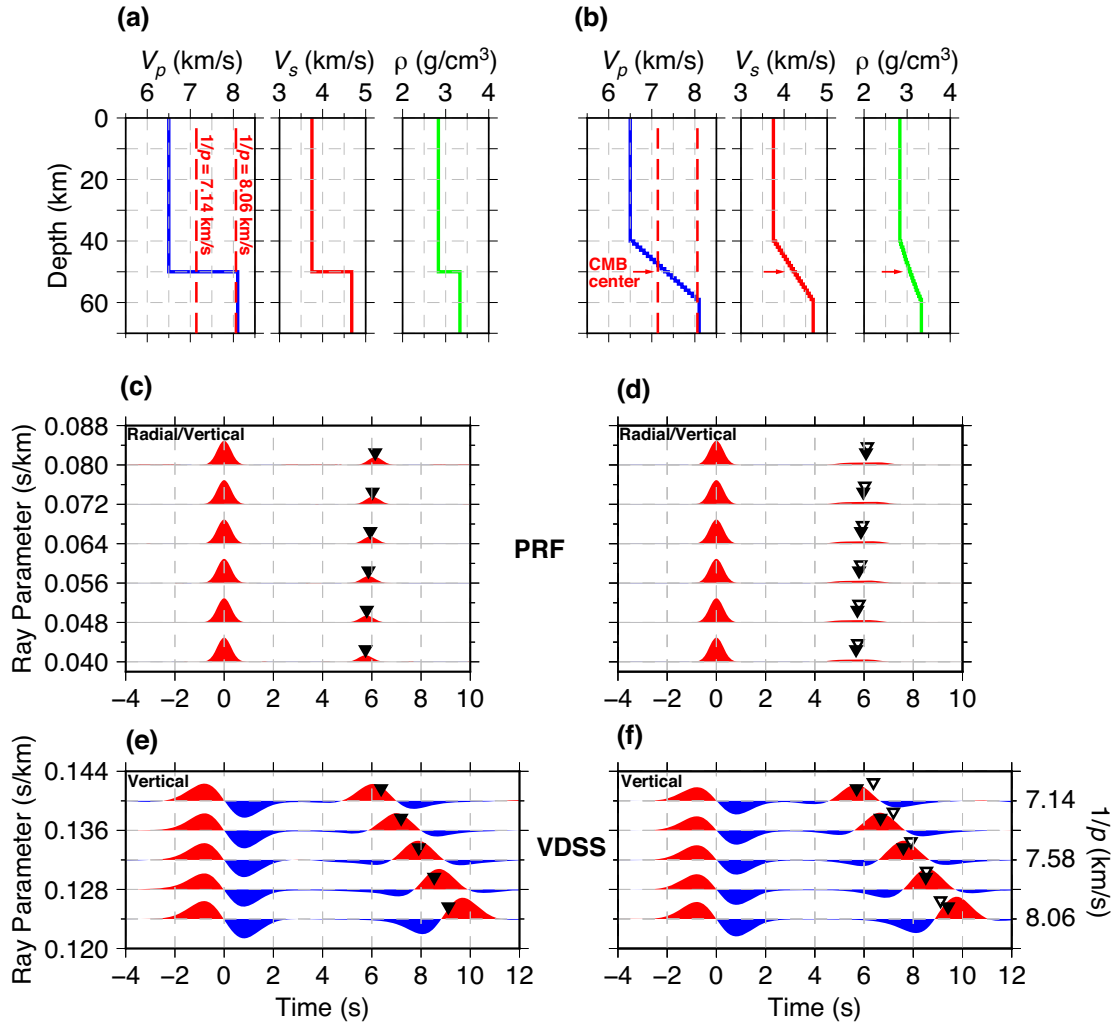


Figure 9. Synthetic RF and *SsPmp* waveforms with different ray parameters for different CMB thicknesses but the same total velocity change across the CMB. (a and b) 1-D models in which the CMB is represented by either a discontinuity in V_p , V_s and density or by a 20 km-thick transition zone. (c and d) Synthetic PRFs (radial components deconvolved by vertical components) computed with the models in (a) and (b), and ray parameters ranging from 0.04 to 0.08 s km⁻¹. Black triangles are theoretical Pms traveltimes for a conversion depth at 50 km, that is, at the abrupt discontinuity (model a) or at the centre of the transition zone (model b). Open triangles in (d) show the theoretical arrival times in model a, offset vertically to be visible. (e and f) Vertical-component synthetic *SsPmp* waveforms computed with models in (a) and (b), and ray parameters ranging from 0.124 to 0.140 s km⁻¹. The corresponding turning velocities ($1/p$) are labelled on the right. Black triangles in (e) are theoretical T_{VDSS} for reflections at the discontinuity, and in (f) are theoretical T_{VDSS} computed for reflections at the depth where $V_p = 1/p$. Open triangles in (f) show T_{VDSS} in (e), offset vertically to be visible. Note that theoretical T_{VDSS} in (e) and (f) form distinct moveout curves with respect to p , and that Φ_{VDSS} varies with p .

earlier than predicted T_{VDSS} (Fig. 10f) and could be easily misinterpreted as *SsPmp* due to its high amplitude (~twice the amplitude of *SsPmp*). We interpret this strong *P* arrival as a superposition of *SsPbp* and *SsbPmp* that arrive at approximately the same time. As the velocity contrast at the basin–basement interface increases, the *S*-to-*P* reflection at the interface becomes stronger, further increasing *SsbPmp* amplitude. If we increase the CMB depth to 60 km, the temporal separation between *SsPbp* and *SsbPmp* will increase so that they appear as two distinct arrivals (Supplementary Information Fig. S1), thereby supporting our interpretation that the strong *P* phase at ~4.5 s in the low- V_p case is the superposition of *SsPbp* and *SsbPmp*. In summary, when basin velocity is relatively low and basin thickness is large enough (so that the dominant frequency component of the incident wave is sensitive to the basin), *SsPbp* and *SsbPmp* become the dominant *P* phases and can be easily misidentified as *SsPmp*, which may cause significant errors in Moho depths inferred from T_{VDSS} .

The above examples show that *SsPmp* amplitude is strongly controlled by basin velocity (Figs 10a and b, e and f), which we interpret as a result of reduced *S*-to-*P* reflection coefficient at the virtual source in the presence of sedimentary basins. To further validate our interpretation, we calculate the peak of *P*-component envelope functions in a 4 s time window around the theoretical T_{VDSS} as a proxy for A_{VDSS} for models with basin V_s varied from 1.45 to 3.75 km s⁻¹ (Fig. 10i; sedimentary layer thickness and ν remain fixed at 6 km and 0.25). We also calculate the *S*-to-*P* reflection coefficient at the free surface for a half-space (i.e. no basin) for the same V_s range and $\nu = 0.25$ (Fig. 10i). A comparison of the two values shows near equivalence for $V_s > 2.25$ km s⁻¹, suggesting the *S*-to-*P* reflection ratio is a primary control on A_{VDSS} (Fig. 10i). As V_s decreases below 2.25 km s⁻¹, A_{VDSS} becomes significantly more variable probably due to interference of *SsPmp* with sedimentary

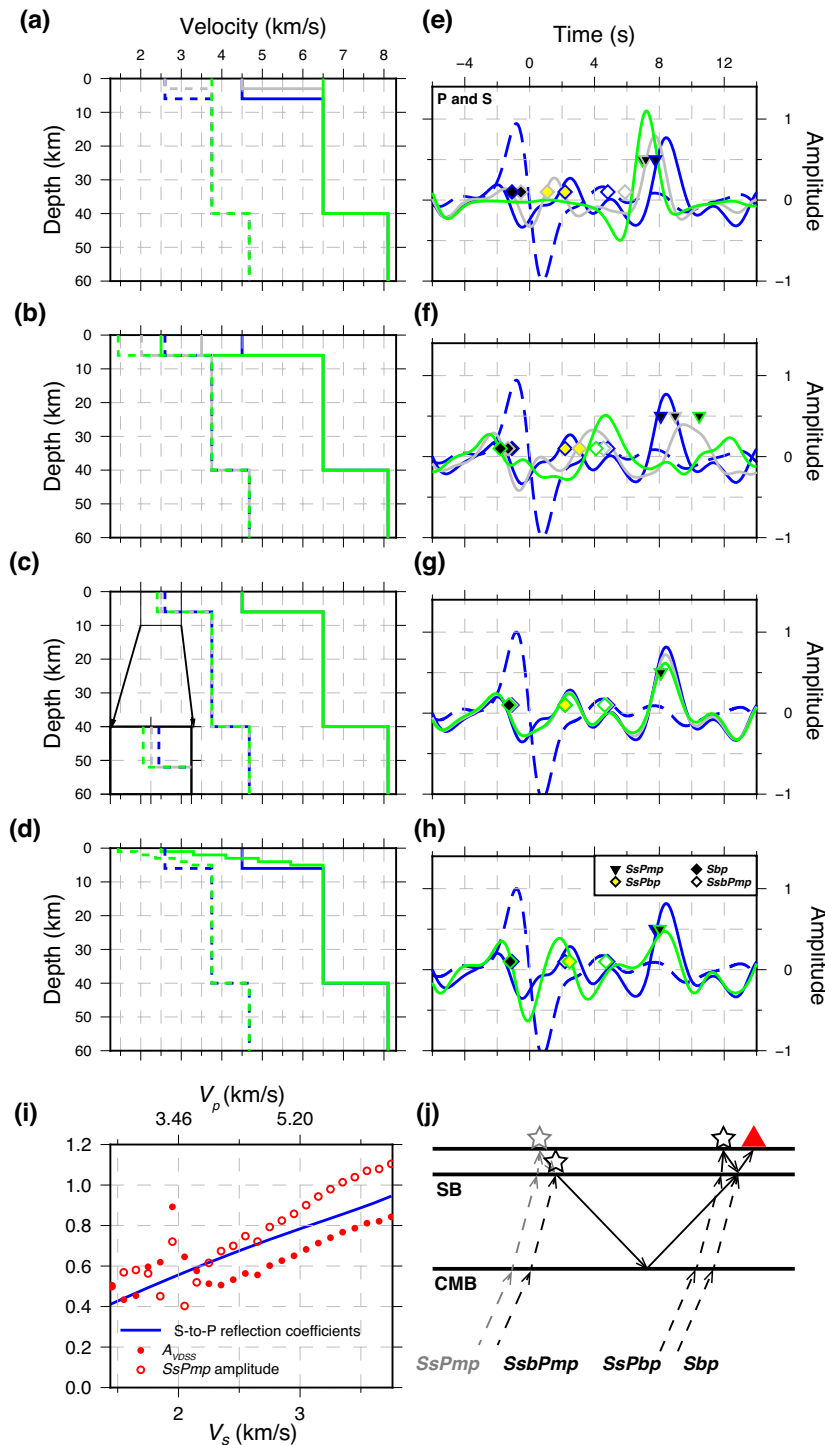


Figure 10. *SsPmp* waveforms for different sedimentary basins on top of the same crust with 40 km-deep CMB. (a–d) V_p (solid lines) and V_s models (dashed lines). Blue lines: reference model, 6 km-thick basin ($V_p = 4.5 \text{ km s}^{-1}$, Poisson's ratio $\nu = 0.25$) above basement ($V_p = 6.5 \text{ km s}^{-1}$, $\nu = 0.25$) and a CMB discontinuity at 40 km depth above $V_p^{\text{um}} = 8.1 \text{ km s}^{-1}$, $\nu = 0.25$. In (a), sedimentary thickness varies: 6 km (blue), 3 km (grey) and 0 km (no sedimentary layer, green). In (b), sedimentary V_p varies: 4.5 km s^{-1} (blue), 3.5 km s^{-1} (grey) and 2.5 km s^{-1} (green). V_s is varied simultaneously to keep Poisson's ratio at 0.25. In (c), sedimentary Poisson's ratio varies: 0.25 (blue), 0.275 (grey) and 0.3 (green), with constant sedimentary $V_p = 4.5 \text{ km s}^{-1}$ (see enlarged inset). In (d), the velocity gradient varies: constant velocity (blue), stack of six 1 km-thick layers with V_p linearly increasing with depth from 1.5 to 6.5 km s^{-1} (green). (e–h) Solid curves: *P*-component synthetic waveforms for velocity models (a)–(d). Dashed blue curves: *S*-component synthetic waveforms for the standard model. Black triangles with edges coloured corresponding to the velocity models mark theoretical arrival times of *Sbp*, *SsPbp* and *SsbPmp* phases. (i) Effects of sedimentary basin on *SsPmp* amplitude. Red solid circles: A_{VDSS} for models with sedimentary V_s ranging from 1.45 to 3.75 km s^{-1} (with fixed basin thickness of 6 km and $\nu = 0.25$, i.e. part b). Red empty circles: *SsPmp* amplitude for the same models. Blue curve: *S*-to-*P* reflection coefficient at the free surface for a half-space (i.e. no basin) for the same V_s range and $\nu = 0.25$. Corresponding sedimentary V_p are plotted on the top. (j) Ray paths of *SsPmp*, *SsbPmp*, *SsPbp* and *Sbp*.

phases that become stronger as V_s decreases. Other factors causing smaller discrepancies between A_{VDSS} and S -to- P reflection ratio likely include transmission loss at the basin–basement interface and finite-frequency effects. In Fig. 10(i), we also show the peak of P -component waveforms (as opposed to envelope functions) in the same 4 s time window around the theoretical T_{VDSS} . Comparison of the envelope amplitude (A_{VDSS}) with the waveform amplitude shows similar trends and behaviour, but the discrepancy in absolute value demonstrates again the need to use the more robust envelope definition of A_{VDSS} .

3.3.3 Sedimentary layer V_p/V_s

To test the effects of sedimentary Poisson's ratio on *SsPmp* waveforms, we vary ν from 0.25, 0.275 to 0.3 (Fig. 10c). Increasing ν corresponds to increasing V_p/V_s ratio as

$$V_p/V_s = \sqrt{\frac{2(1-\nu)}{1-2\nu}}. \quad (2)$$

Because we hold basin V_p constant, increasing ν causes basin V_s , thus S -wave incident angle, to decrease. Therefore, the amplitude of *SsPmp* decreases with increasing sedimentary Poisson's ratio (Fig. 10g). Although the range of ν used here is quite large and covers most sedimentary rocks (Brocher 2005), the effect of basin ν on *SsPmp* amplitude is rather limited (Fig. 10g). Because T_{VDSS} is only related to V_p in the crust (eq. 1), despite changes of *SsPmp* amplitude, *SsPmp* arrives at identical times in all three cases (Fig. 10g).

3.3.4 Sedimentary layer velocity gradient

To test how vertical velocity gradient in sedimentary basins affects *SsPmp* waveforms, we compare results for a constant-velocity basin (homogeneous case) with results from a basin formed of six 1 km-thick layers with velocity linearly increasing from 2.5 km s⁻¹ at the surface to 6.5 km s⁻¹ at the basin–basement interface (gradient case; Fig. 10d; cf. Parker *et al.* 2016). Although both basins have the same average velocity, our synthetics show that *SsPmp* amplitude is further reduced in the gradient case (Fig. 10h), presumably because lower V_s at the free surface decreases the S -to- P reflection coefficient for high-frequency components, which are more sensitive to shallow structure, at the virtual source. Because sedimentary basins commonly have velocities increasing with depth, this example explains why observed *SsPmp* amplitude can be lower than modelled for homogeneous sedimentary layers with equivalent delay times.

3.4 Sensitivity to lateral heterogeneity in the mantle

Yu *et al.* (2016) discussed the effects of local-scale lateral heterogeneity in the crust and the uppermost mantle on traveltime differences between the upgoing S legs of *SsPmp* and Ss , which in turn result in deviations in T_{VDSS} from the 1-D assumption (eq. 1). A simple example would be the juxtaposition of thick Archean lithosphere (>200 km) and Phanerozoic lithosphere of normal thickness (~80 km), putatively the case at the boundaries of the Ordos Block (Fig. 4; Yu *et al.* 2012). We would like to quantify the effects of such lateral heterogeneity on *SsPmp* waveforms.

We create a 2-D model with a constant crustal thickness of 40 km, in order to explore only the effect of mantle heterogeneity on *SsPmp* waveforms (Fig. 11a). The lithosphere comprising both the crust and lithospheric mantle is 80 km thick at each end, and 200 km thick

in the centre of the model. The 200 km-thick lithosphere linearly shallows to 80 km across 200 km, forming two straight ramps dipping at 31° (Fig. 11a). The velocity of the lithospheric mantle is ~5% faster than the asthenosphere (Fischer *et al.* 2010). We use the SPECFEM2D software package to compute the surface-recorded seismograms of a plane S wave with ray parameter 0.1275 s km⁻¹ (incident angle of 35° in the asthenosphere) incident on the bottom left of the model (Komatitsch & Vilotte 1998; Komatitsch *et al.* 2001; Tromp *et al.* 2008). Despite a constant crustal thickness in our model, the synthetic seismograms show significant variation of T_{VDSS} across the recording array (Fig. 11b). On the far left of the array, where both virtual sources and receivers are located above thin lithosphere, *SsPmp* arrives at ~6.9 s (Fig. 11b), in agreement with the 1-D prediction. When the receiver is located at the edge of the area with thick lithosphere (blue triangle in Fig. 11a), with the corresponding virtual source (blue star in Fig. 11a) above thinner lithosphere, T_{VDSS} increases to ~8 s, significantly later than the 1-D model prediction (blue traces in Figs 11b and c). In the centre of the area with thick lithosphere, where receivers and corresponding virtual sources are both in areas with thick lithosphere, observed T_{VDSS} returns to the time predicted for the 1-D model (black virtual source, receiver and traces in Figs 11a–c). At the right edge of the area with thick lithosphere, where the receivers are located above thinner lithosphere than their virtual sources, T_{VDSS} rapidly decreases to ~5.5 s, significantly earlier than the 1-D prediction (red triangle, star and traces in Figs 11a–c). If this observed T_{VDSS} is used to infer crustal thickness with a 1-D assumption, the 1.5 s difference from the theoretical T_{VDSS} will cause ~8 km error in Moho depth. The explanation for the anomalous T_{VDSS} is that when the upgoing S -wave leg of Ss travels through a velocity anomaly that is not present on the path of *SsPmp* (Fig. 11d), the traveltime difference between the two ray paths deviate from predictions made by assuming 1-D Earth structure, and the residual will be included in the observed T_{VDSS} . Therefore, when applying VDSS to data collected in areas with significant variation in lithosphere thickness, it is vital to correct for S traveltime residuals before converting observed T_{VDSS} to crustal thickness. A practical approach to this correction is presented in Part 2.

Another interesting feature of our synthetic seismograms is that the Ss observed above the right margin of the thick lithosphere is significantly stronger than that observed along the rest of the profile (red traces in Figs 11b and c). The increase in Ss amplitude cannot be due to crustal structure as the crustal thickness is uniform along the profile. Instead, the lithosphere ramp on the right edge of the thick lithosphere causes energy to be focused (multipathing effect; see Part 2). Note that we apply no normalization to the traces in Figs 11(b) and (c). If the traces are normalized with the peak Ss amplitude of each trace, a common practice in VDSS data processing, the red trace at the right edge of thick lithosphere will have a low-*SsPmp* amplitude of 0.83 due to its high- Ss amplitude. In contrast, the blue trace at the left edge of thick lithosphere will have a high-*SsPmp* amplitude of 1.23. Thus, lateral heterogeneity in the mantle can affect A_{VDSS} , as well as T_{VDSS} , by changing the energy distribution across the incident S -wave front.

4 DISCUSSION

The nature of *SsPmp* as a post-critical, wide-angle reflection phase brings both advantages and disadvantages in using T_{VDSS} to infer Moho depth. A clear advantage of *SsPmp* compared with narrow-angle phases (e.g. Pms used in PRF) is its high amplitude, which

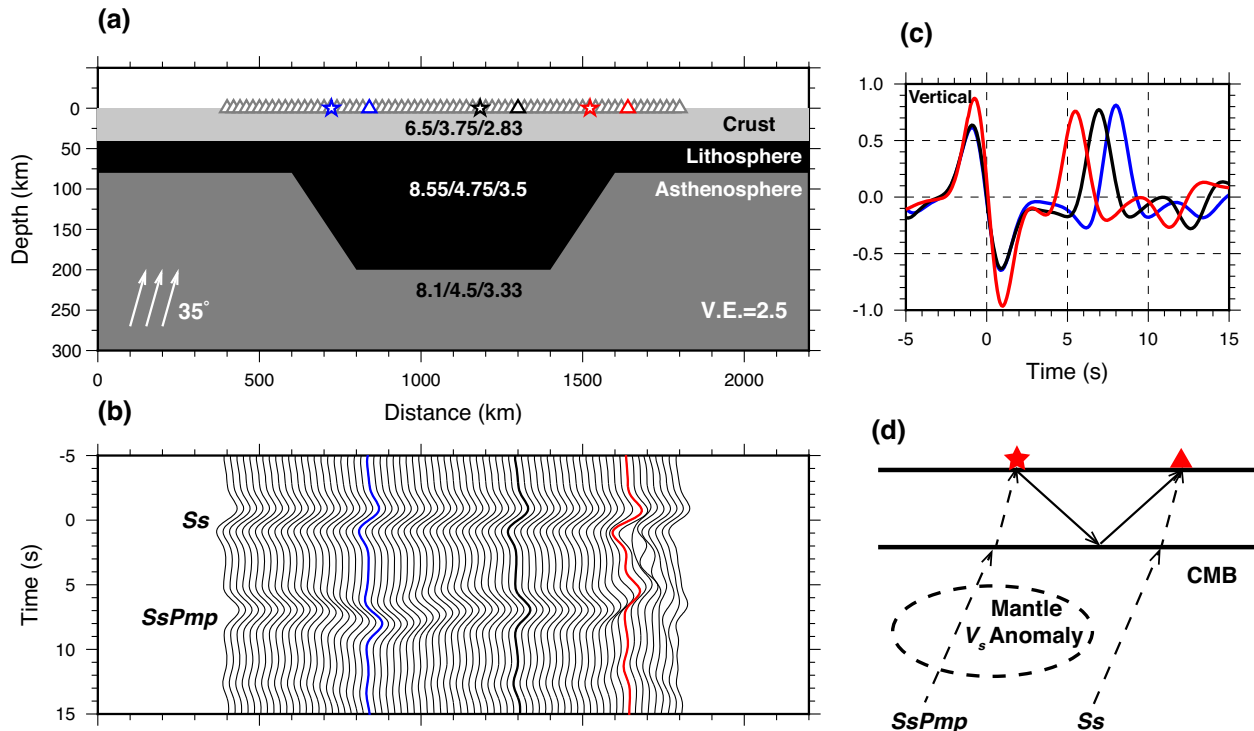


Figure 11. Effects of lateral variation in mantle V_s on T_{VDSS} . (a) 2-D model used to compute synthetic seismograms with a vertical exaggeration of 2.5. Light grey: crust. Black: lithospheric mantle. Dark grey: asthenosphere. Material properties of model layers are listed as V_p (km s⁻¹)/ V_s (km s⁻¹)/ ρ (g cm⁻³). Incident angle of the plane S wave is 35° and the dip of the lithosphere–asthenosphere boundary is 31°. The seismograms of the stations highlighted with blue, black and red are shown in (b) and (c). Stars show the virtual sources of the stations highlighted with the same colours. (b) Vertical-component synthetic seismograms (*not* P or S components as shown in all other figures) recorded by stations in (a). Black, blue and red traces: traces recorded by highlighted stations in (a) and coloured accordingly. Note the significant lateral variation of T_{VDSS} despite homogeneous crustal thickness. Amplitudes are relative to largest amplitude on plot (*not* normalized by peak S_s amplitude of each trace). (c) Vertical-component synthetic seismograms recorded by highlighted stations in (a) and coloured accordingly. (d) Schematic showing V_s anomaly in the mantle affects T_{VDSS} by changing the relative traveltimes of S_s and the S legs of $SsPmp$.

makes it possible to constrain crustal thickness with a single record. In addition, the long ray path of $SsPmp$ in the crust causes T_{VDSS} to have a stronger dependence on ray parameter (or higher ‘move-out’) than Pms , making it feasible to use the moveout of $SsPmp$ to simultaneously constrain crustal thickness and crustal average V_p (Kang *et al.* 2016; Parker *et al.* 2016). In addition, given a uniform azimuthal distribution of earthquakes, it is possible to measure crustal thickness on a circle with ~60 km radius centred at the receiver, a much broader range than the CMB depth sampled by PRFs that is usually within 15 km from the receiver. Moreover, post-critical reflection causes a phase shift Φ_{VDSS} between $SsPmp$ and S_s , which can potentially be used to determine V_p^{lc} and V_p^{um} , though this inference requires particularly high-quality waveform data (see Part 2).

Disadvantages of the long crustal ray path of $SsPmp$ include the sensitivity of $SsPmp$ to lateral variation over a wide area, both in basins near the surface and at mantle depths. For instance, Fig. 10 shows that sedimentary basins at virtual sources can significantly reduce $SsPmp$ amplitude, making it difficult to measure T_{VDSS} . Sedimentary basins may also generate phases ($SsPbp$, $SsbPmp$, etc.) that are stronger than $SsPmp$ and can easily be misidentified as $SsPmp$, thus causing large errors in inferred Moho depths (Fig. 10; though also offering the ability to study basin structure). Dense 1-D or 2-D arrays that cover both basins and adjacent exposed basement are key to avoiding such mistakes, because the gradual transition in waveforms allows recognition of sedimentary effects. In a second example, Fig. 11 shows the effect of heterogeneous mantle velocity

structure on $SsPmp$ amplitude and T_{VDSS} , which can be partially overcome by application of ‘static’ corrections given data collected by a dense 2-D array (Yu *et al.* 2016). In general, future work using $SsPmp$ to study crustal properties should leverage the power of dense arrays to alleviate challenges posed by its long ray path in the crust.

When the CMB is a transition zone, $SsPmp$ is a turning wave with post-critical reflection at the depth where V_p reaches the turning velocity ($1/p$). Because $SsPmp$ is high amplitude even for a transitional CMB, it is possible to use $SsPmp$ with different ray parameters, which turn at different depths, to constrain the interior structure of the CMB (Fig. 9). In contrast, conversion phases such as Pms (and pre-critical reflection phases) may be too weak to image a transitional CMB. In areas with poorly defined or absent PRF Moho, such as the Colorado Plateau, analysis of post-critical $SsPmp$ may reveal more details of the CMB.

The Ordos Plateau is overlain by thick west-dipping Phanerozoic strata, with the age of exposed rocks decreasing westwards (Yang *et al.* 2005; Xia *et al.* 2017). The virtual source of JB09 is west of the virtual source of JB13 (Fig. 4a), so is above younger, thus likely lower-velocity sedimentary rocks. A wide-age range of sedimentary rocks may also cause significant velocity gradient in the basin. These factors likely explain the significantly reduced $SsPmp$ amplitude at JB09 compared to JB13. In contrast to the change in $SsPmp$ amplitude between JB09 and JB13, the change in Φ_{VDSS} between JB13 and FY03 may have a more complicated origin. As shown in Figs 5 and 6, elevated V_p^{lc} and V_p^{um} reduce Φ_{VDSS} and may cause

$\Phi_{\text{VDSS}} \approx 0^\circ$ at FY03. In addition, if the CMB is dipping from the virtual source towards the station, the incident angle of *SsPmp* at the Moho is increased, thus increasing ray parameter p and decreasing Φ_{VDSS} (Fig. 7). Because the virtual source of FY03 is closer than station JB13 to the actively extending Shanxi Graben System (Fig 4a), the crust at the virtual source is likely thinner than at the station, causing a dipping CMB that may also contribute to the very small Φ_{VDSS} at FY03. Quantitative analysis of change in *SsPmp* amplitude and Φ_{VDSS} across the Ordos Plateau is beyond the scope of this paper and will be presented elsewhere.

5 CONCLUSION

We use synthetic seismograms to show that the phase shift of *SsPmp* relative to *Ss* (Φ_{VDSS}) decreases as lower-crust P -wave velocity (V_p^{lc}), upper-mantle P -wave velocity (V_p^{um}), or ray parameter increase, making it possible to constrain V_p^{lc} and V_p^{um} with observed *SsPmp* waveforms. Amplitude of *SsPmp* (as a proxy for A_{VDSS}) is insensitive to CMB thickness but is strongly affected by near-surface velocity at the virtual source. If the CMB is a velocity-gradient zone, *SsPmp* is a turning wave within the CMB, with turning depth determined by the ray parameter, thereby offering the potential to image the CMB in areas where methods based on conversion and pre-critical reflection phases fail to show a clear Moho. We also show that T_{VDSS} is affected not only by crustal thickness, but also by crustal and upper-mantle heterogeneity, which requires array data for proper analysis.

ACKNOWLEDGEMENTS

TL is supported by a Stanford Graduate Fellowship. We thank all the people who were involved in the deployment of the Peking University temporary array in the Ordos Plateau.

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SUPPORTING INFORMATION

Supplementary data are available at [GJI](#) online.

Figure S1. (a) The same as Fig. 10(b). (b) V_p and V_s models with the same sedimentary layer thickness (6 km), V_p (4.5 km s^{-1}) and Poisson's ratio (0.25), but with CMB depth increased from 40 km (green curve) to 60 km (magenta curve). The green model is the same as the green one in (a). (c) The same as Fig. 10(f). (d) Solid curves: P -component synthetic waveforms calculated from velocity models in (b) and coloured as in (b). Dashed green curves: S -component synthetic waveforms computed with the green model in (b).

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